# *Montana Comprehensive Assessment System (MontCAS, Phase 2)*

## *Criterion-Referenced Test (CRT)*

## Common Constructed-Response Item Release
## Mathematics, Grade 8

2006

OPI

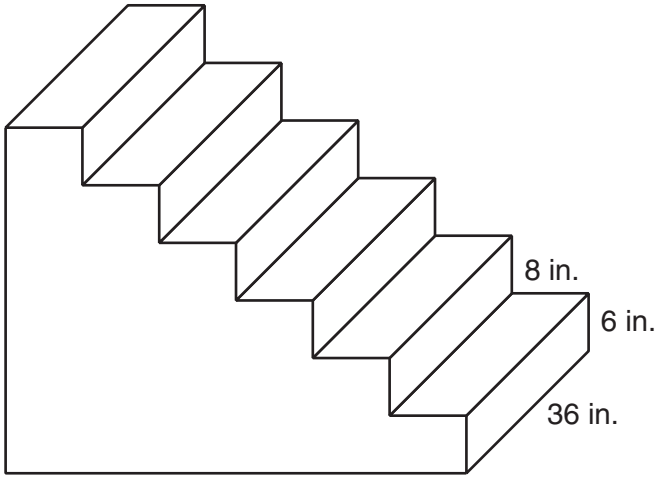
OFFICE OF PUBLIC INSTRUCTION

# Mathematics

## Session 1 (Calculator)

**You may use a calculator during this session.**

25. The staircase shown below is made entirely out of concrete.




There are 6 steps. Each step has the same dimensions.

a. What is the height, in feet, of the entire staircase?

b. What is the volume of the bottom step in cubic feet? Show or explain how you found your answer.

c. What is the volume of the entire staircase in cubic feet? Show or explain how you found your answer.

## Scoring Guide

| Score | Description |
|---|---|
| **4** | 4 points |
| **3** | 3½ points<br>OR<br>3 points with at least ½ point earned in each part |
| **2** | 2 – 3 points |
| **1** | ½ – 1½ points<br>OR<br>Student shows minimal understanding of volume of a compound solid |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

**Scoring Notes:**

Part a: 1 point for correct answer [**3 (ft.)**]
OR
½ point for correct answer in inches (36 inches - units necessary)

Part b: 2 points for correct answer [**1 (cubic ft.)**] with work shown.
OR
1 point for correct answer with no work
or
for finding 1728 (cubic in.) with work shown
or
for incorrect answer given in feet with correct strategy but minor error in work
or
for finding 6 (cubic ft.) – the bottom step being the entire bottom of the staircase
OR
½ point for incorrect answer given in inches with correct strategy but minor error in work
or
for finding 1728 (cubic in.) with no work shown
or
for finding 1728 cubic feet with work shown
or
for finding 10,368 (cu. in.) – the bottom step being the entire bottom of the staircase

Part c: 1 point for correct answer [**21 cubic ft.**] with work shown
OR
½ point for incorrect answer with minor computation error in work
or
for correct answer with no work.

**Notes:**

- If an error in calculation (i.e. incorrect conversion of 8 inches to ? foot) is repeated in more than one part only take off once.
- Read along with the responses. If an error is made in one part, do not penalize for that error being carried through to other parts.

**Sample Response:**

Part a: $6 + 6 + 6 + 6 + 6 + 6 = 36$; $36 \div 12 = 3$ feet

Part b: $6 \times 8 \times 36 = 1728$ cubic inches
1 cubic foot = 12 inches × 12 inches × 12 inches = 1728 inches
$\therefore$ The bottom step is 1 cubic foot.

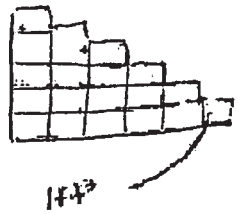
Part c: Recognize that each step has the same depth and width but is higher than the previous step by 6 in. The volume of each step is one cubic foot more than the previous step. So add: $1 + 2 + 3 + 4 + 5 + 6 = 21$ cubic feet. In other words, treat the staircase as a set of blocks the size of the first step. There are twenty-one such blocks in the drawing.
OR
View each step as a triangular prism with volume $\frac{1}{2} \times 8 \times 6 \times 36 = 864$ cubic inches or .5 cubic feet. There are 6 steps for a total volume of $.5 \times 6 = 3$ cubic feet. Now consider the base of the staircase. It too is a triangular prism with a height of 3 feet, a depth of 4 feet (each step is 8 inches deep, times 6 steps, equals 48 inches or 4 feet) and width 3 feet. The volume of the staircase is $\frac{1}{2} \times 3 \times 4 \times 3 = 18$ cubic feet. So the total volume of the stairs and staircase combined is $3 + 18 = 21$ cubic feet.

**Score Point 4**

**Sample 1**

a. 3 Feet $6+6+6+6+6+6=36 \div 12=3$ feet

b. $1\ ft^3$ $\frac{2}{3} \cdot \frac{1}{2}=\frac{2}{6}$ or $\frac{1}{3}$ and $\frac{1}{3} \cdot 3=1$

c.  $21\ ft^3$

$1\ ft^3$

there is 21 blocks

that are $1\ ft^3$ so

the volume of the

entire staircase is

$21\ ft^3$

**Score Point 4**

**Sample 2**

a) 3 feet

b) volume = length × width × height

$V = 9 \times 6 \times 36$

$V = \frac{3}{4} \times \frac{2}{3} \times 3$ } convert to feet

$V = 1\ ft^3$ = volume of 1st stair } solve

c) # of steps = 6

# of blocks under steps = 15

total # of blocks = 21

total volume = Volume of 1 block × 21

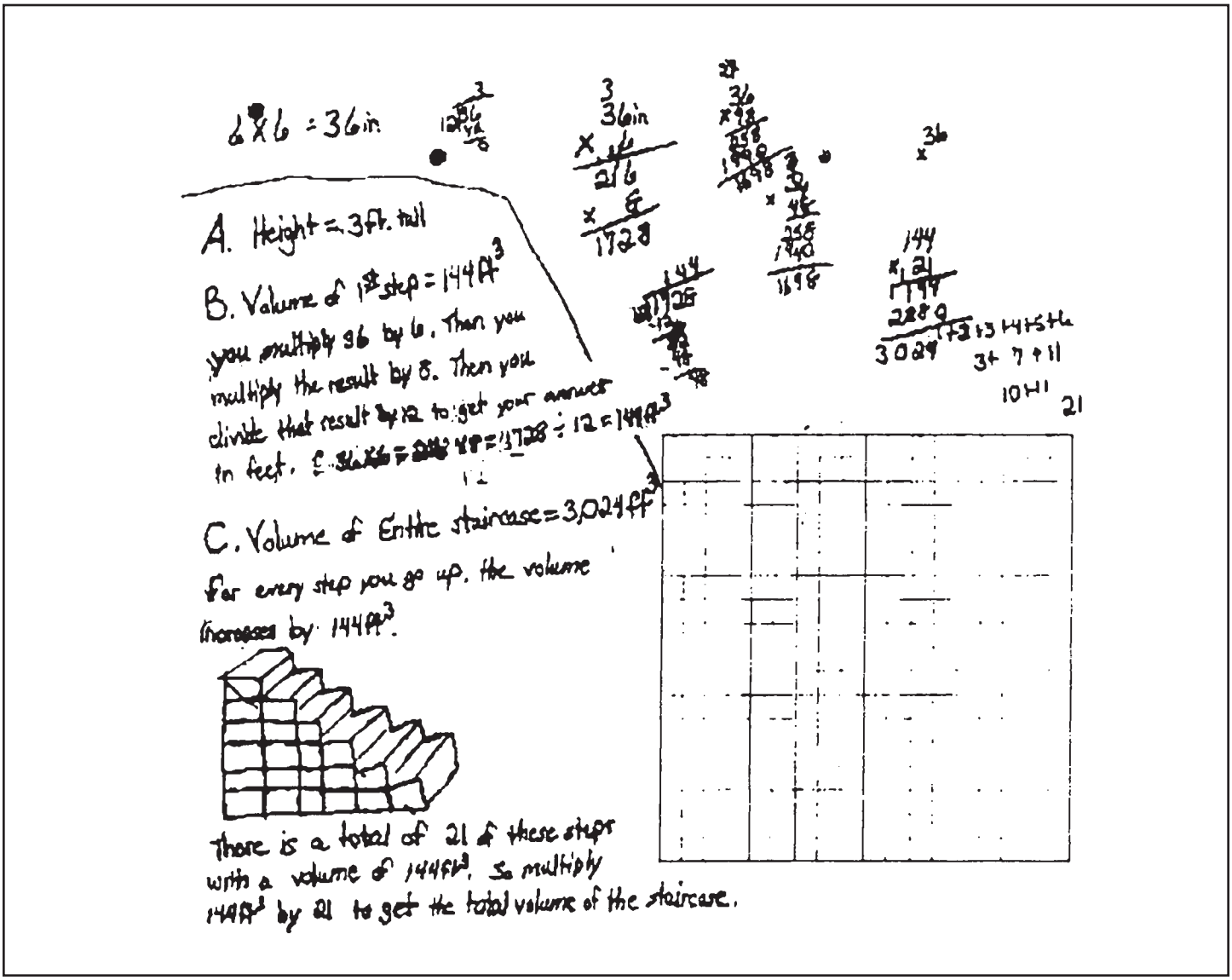
$V = 1ft^3 \times 21$

$V = 21ft^3$ = Volume of staircase

I first found the total number of stair shaped blocks in the staircase. There were 21. I then multiplied the area of 1 block by the number of blocks ($1ft^3 \times 21$). I got $21\ ft^3$.

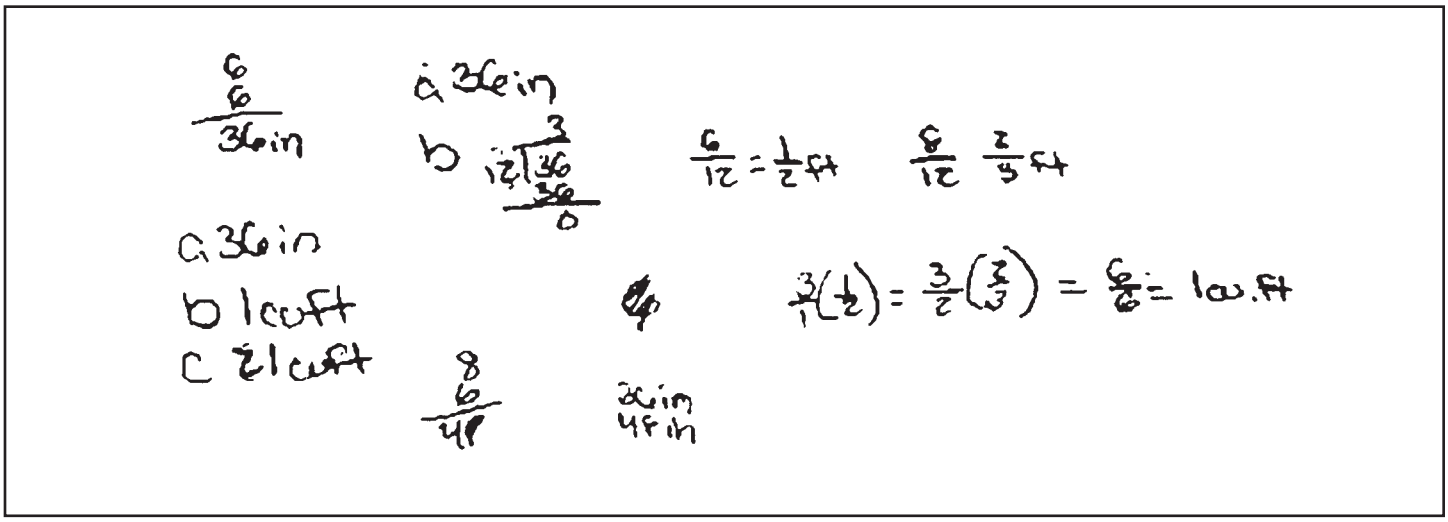
**Score Point 3**

**Sample 1**



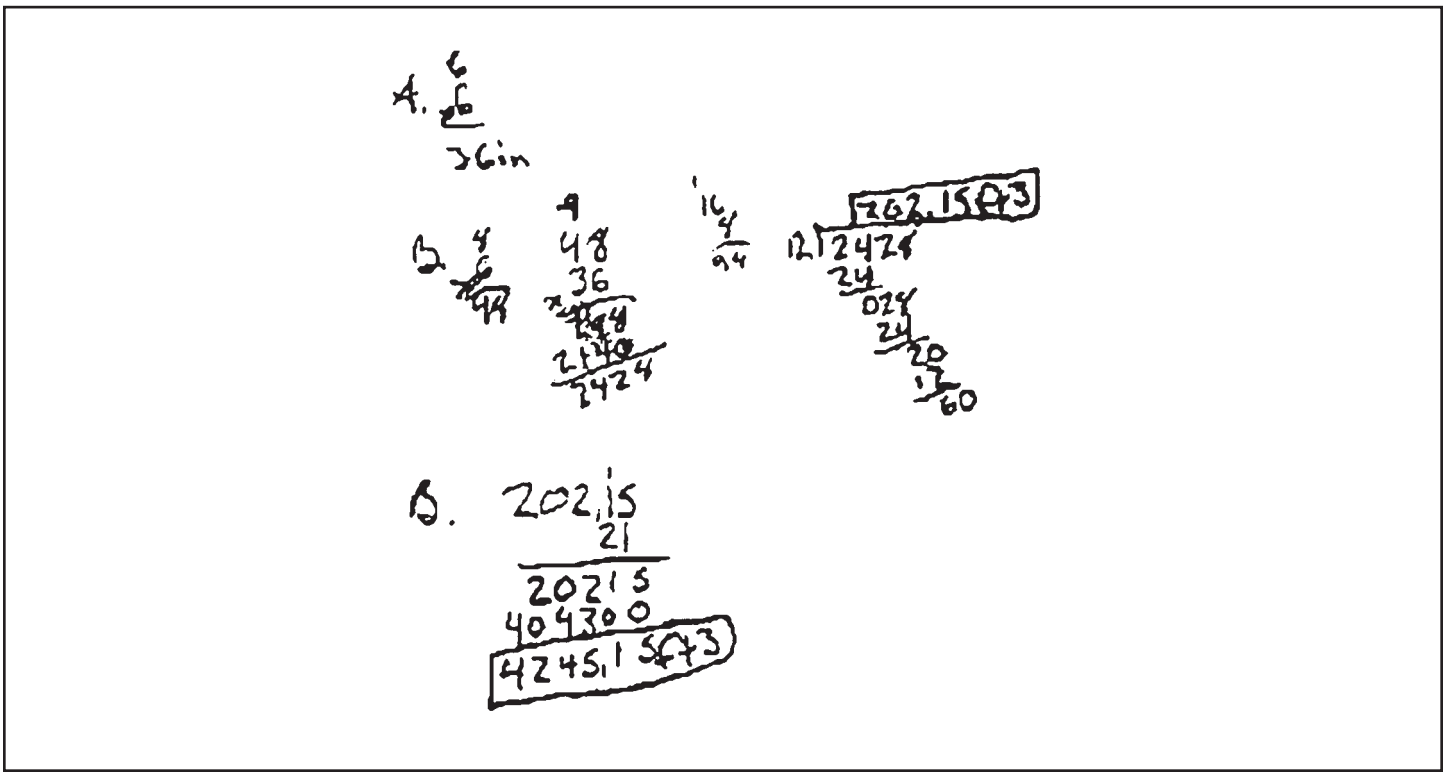
**Score Point 3**

**Sample 2**



**Score Point 2**

**Sample 1**




**Score Point 2**

**Sample 2**

a.) The staircase is three ft. tall

b.) The volume is 0.9ft.$^3$ I found my answer by multiplying length x width x height, so I converted 1in to 0.6 ft, 1in. to 0.5 ft, and 3in. to 3 ft because I needed to find cubic feet and not cubic inches. I multiplied 0.6 ft. by 0.5ft, then multiplied by 3ft. My product came out to be 0.9 ft.$^3$

c.) The volume of the staircase is 16.2 ft.$^3$ because I multiplied 0.9ft.$^3$ (the volume of one stair) by 18 because in the picture there is a space underneath the stairs as they go up, so I ~~thought~~ that the space under the stairs were like more stairs underneath. Then, I counted how many "invisible" stairs there were, and came up with eighteen.

**Score Point 1**

**Sample 1**

a. 36 in.

b. 1'728

c. 60'208

**Score Point 1**

**Sample 2**

A.) 108ft.

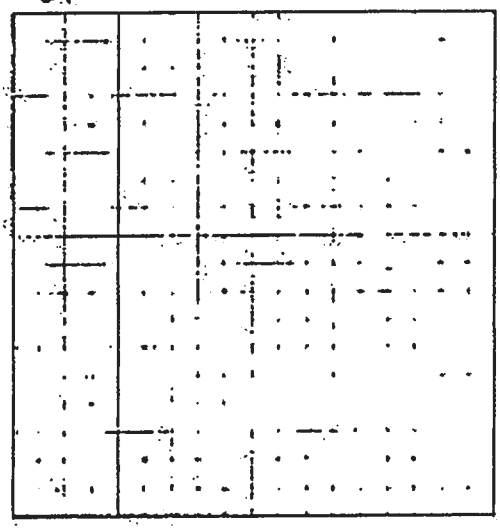
B.) 960ft.$^3$

C.) 20,160 ft$^3$

I multiplied the volume of the bottom step by 21.

960
x 21
960
+19200
20160

B- 36x6x4 = 864
8x6x2 = 96
960



**Score Point 0**

**Sample 1**

a. 40 in. = 3ft. 4in. 40/12 = 3.33

b. .99 cubic ft. 36/3 = 1 ft. * .5 ft * .66 ft

c. 6x21 = 126 in. + 49 in. + 35 in = 210 cubic ft.

**Score Point 0**

**Sample 2**

a.) 10 ft. 6 inches

b.) 3 ft. multiply 12 inches by 3, youll get 3 ft.

c.) 43 ft.

# Mathematics

## Session 3 (No Calculator)

**You may NOT use a calculator during this session.**

68. The following sandwich choices are offered at Victor's Sandwich Bar.

**Victor's Sandwich Choices**

| Bread | Meat | Cheese |
|---|---|---|
| Wheat<br>Rye | Turkey<br>Chicken<br>Beef | American<br>Swiss |

a. How many different sandwich choices are possible if every choice consists of one type of bread, one meat, and one cheese? Show your work or explain how you found your answer.

b. A Deluxe Sandwich consists of one type of bread, **two** meats, and one cheese. How many different Deluxe Sandwich choices are possible? Show your work or explain how you found your answer.

## Scoring Guide

| Score | Description |
|---|---|
| **4** | 4 points |
| **3** | 3 points |
| **2** | 2 points |
| **1** | 1 point<br>OR<br>Minimal understanding of combinations and/or systematic counting in context |
| **0** | Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured. |
| **Blank** | No response. |

**Scoring Notes**

Part a: 2 points for the correct answer [**12**] and work shown or explanation given. Methods may vary. Students can show their work with tree diagrams, organized list, or computation shown below.

$2 \times 3 \times 2$

OR

1 point for the correct answer with insufficient or no work or explanation
or
for correct strategy (tree diagram or organized list showing at least 8 different combinations)

Part b: 2 points for the correct answer [**12 OR 24 – see note**] with sufficient work shown or explanation to indicate correct strategy. Methods may vary. Students can show their work with tree diagrams or organized lists such as:

| | |
|---|---|
| W T/C A | R T/C A |
| W T/C S | R T/C S |
| W T/B A | R T/B A |
| W T/B S | R T/B S |
| W C/B A | R C/B A |
| W C/B S | R C/B S |

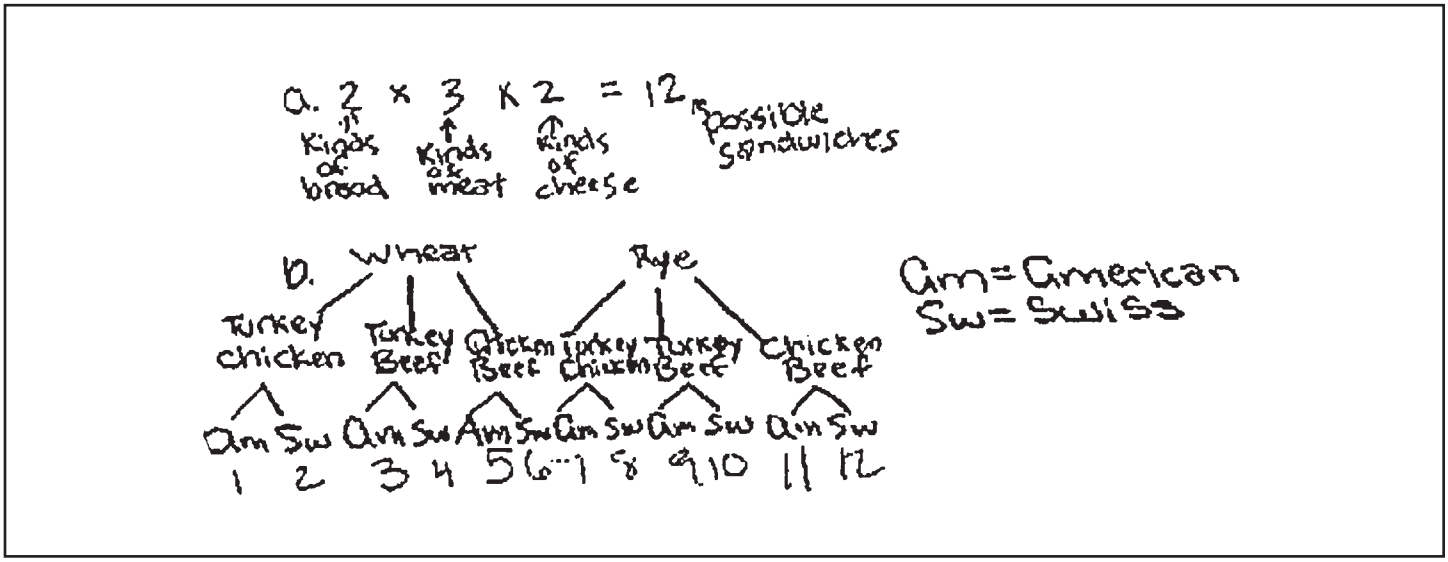
OR

1 point for **either** correct answer with insufficient no work or explanation
or
for correct strategy shown (tree diagram or organized list showing at least 8 different combinations)

**Note for part b**: Students may interpret question as allowing "double meat" as an option, i.e., turkey/turkey, chicken/chicken, beef/beef, without penalty. In this case, there are 24 combinations possible. Since either interpretation is readily defensible, credit for correct answer with insufficient or no explanation should be given for answer of 24 as well as 12.

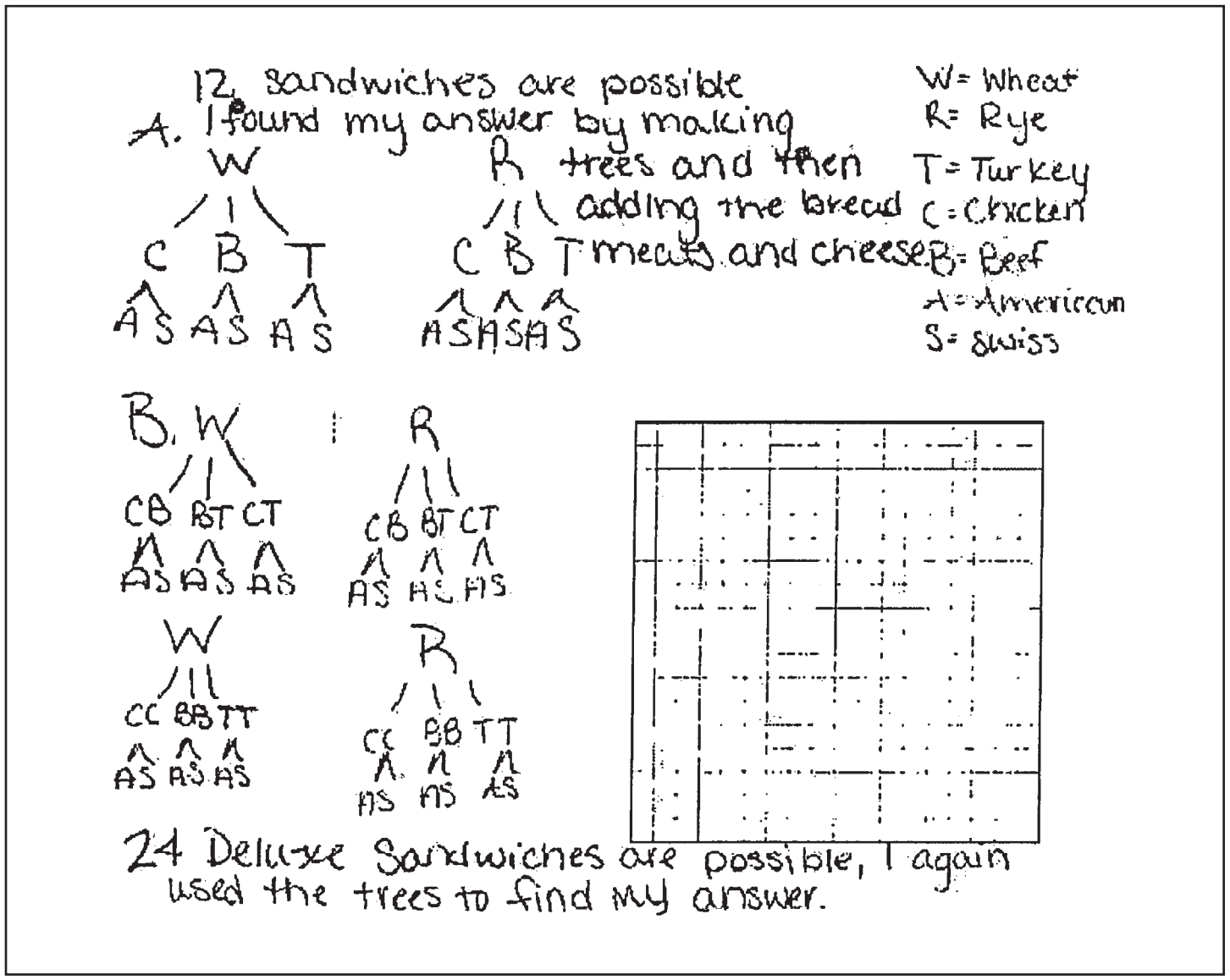
**Score Point 4**

**Sample 1**



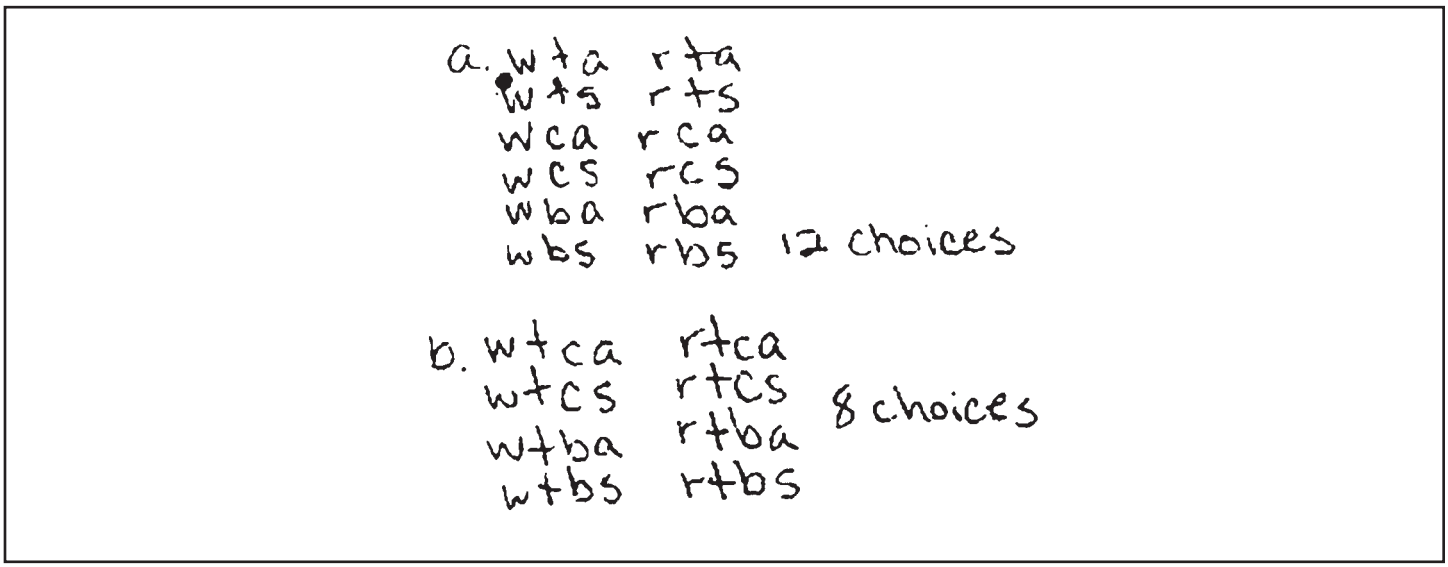
**Score Point 4**

**Sample 2**



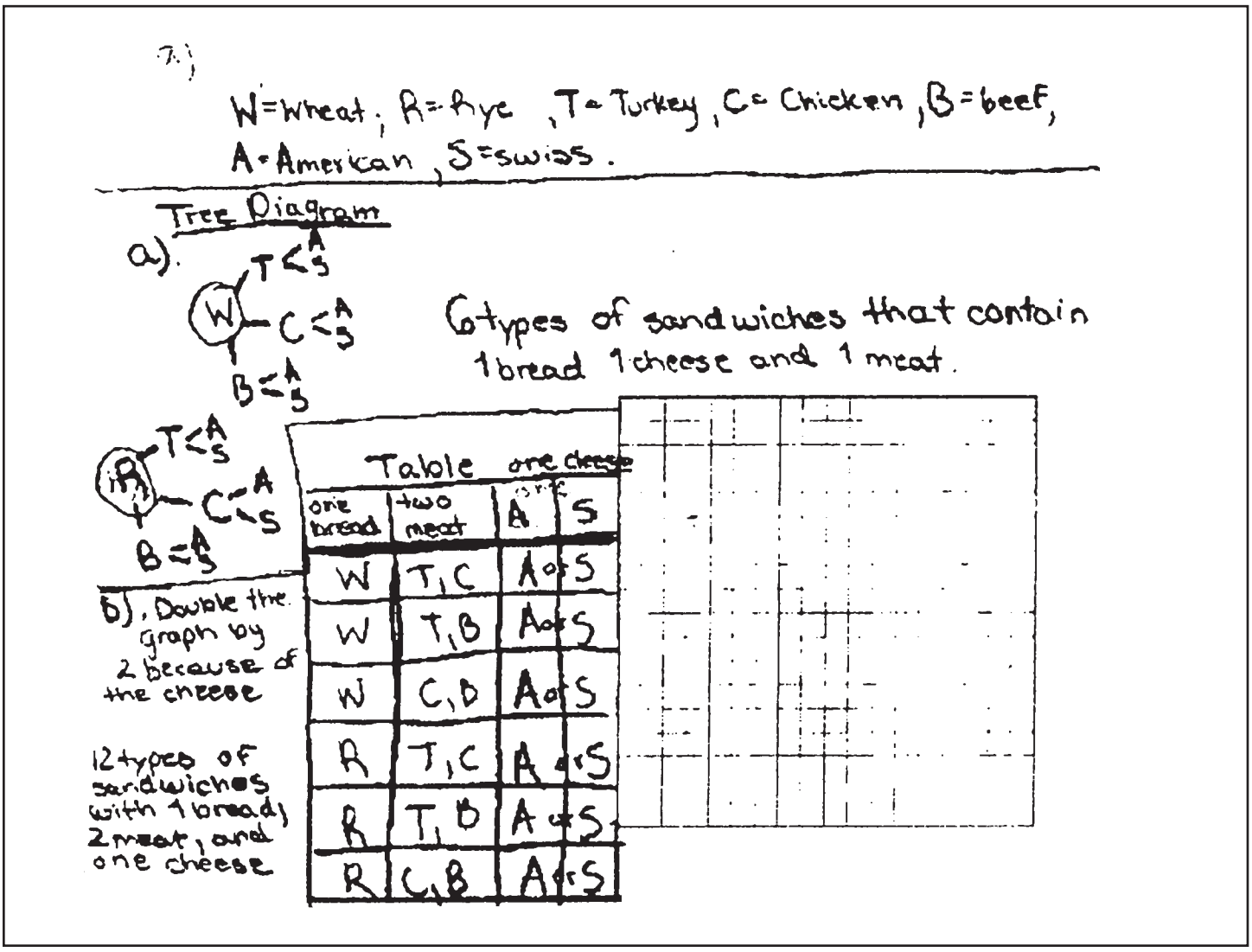
**Score Point 3**

**Sample 1**



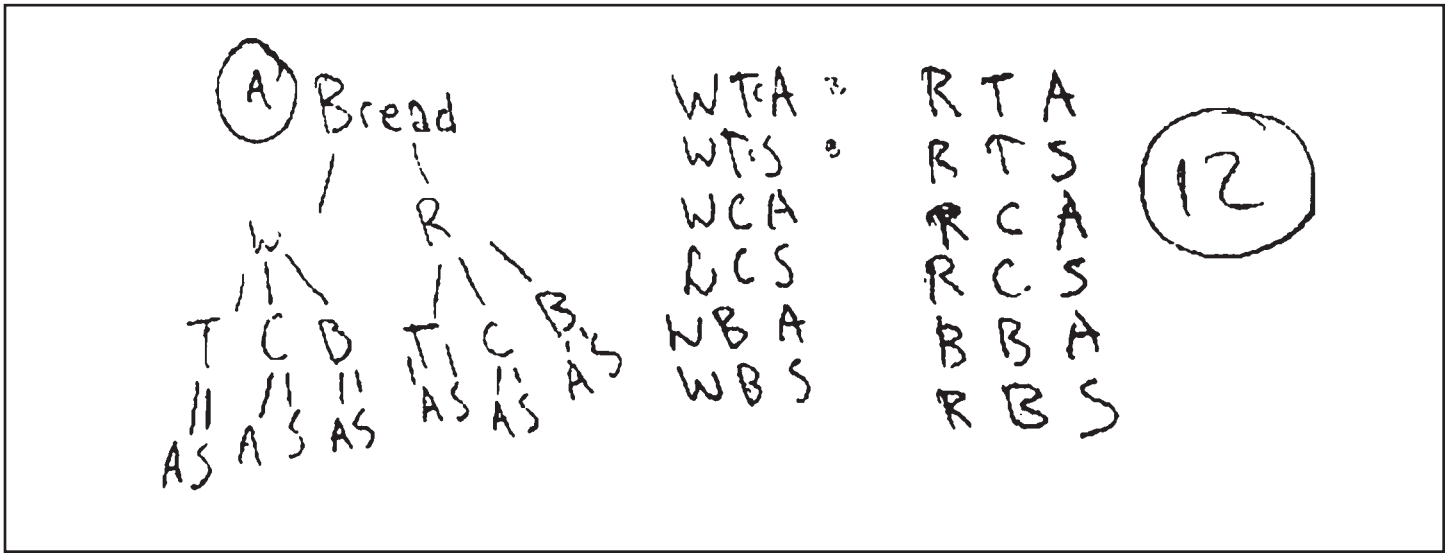
**Score Point 3**

**Sample 2**



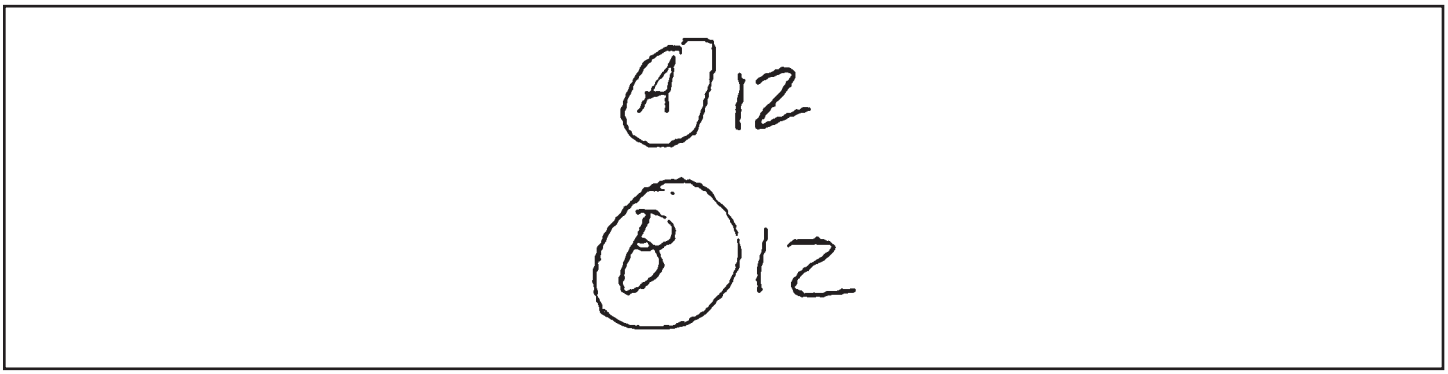
**Score Point 2**

**Sample 1**



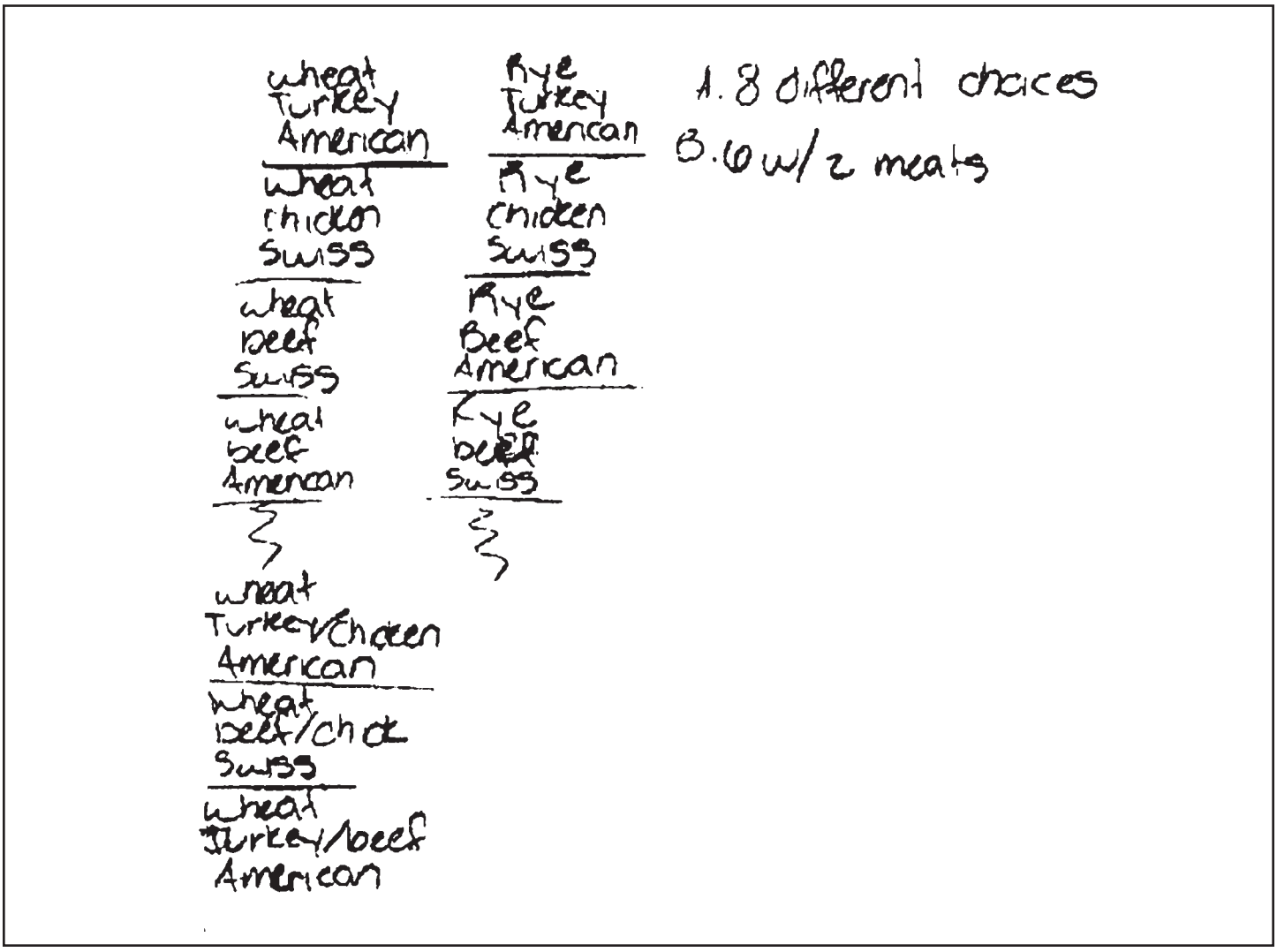

**Score Point 2**

**Sample 2**



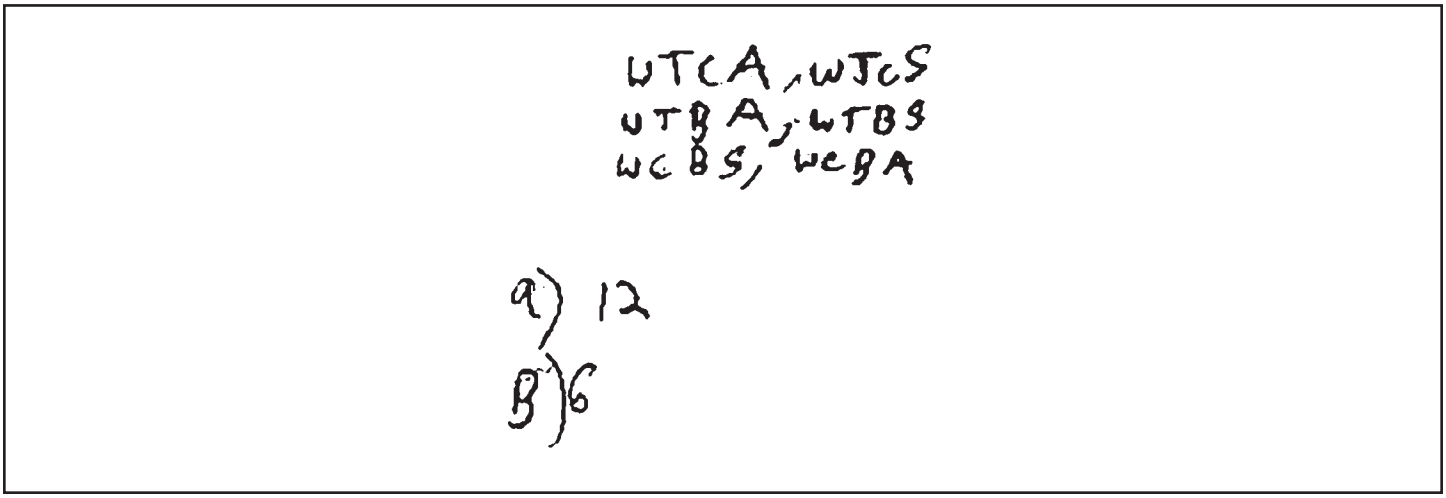
**Score Point 1**

**Sample 1**



**Score Point 1**

**Sample 2**



**Score Point 0**

**Sample 1**

Wheat
Chicken
American

Well the reason I choose them is because I like wheat cause it's good for you. And chicken well it's not all that health

**Score Point 0**

**Sample 2**

a. 10 possible — you take Bread with Turkey and you could have two kinds of cheese so I took that which every different kind

b. 8 possible — I took bread with Turkey and chicken with 1 type of cheese then took Turkey with Beef with one kind of cheese and kept repeating these steps with the each type of bread